5. A population is modeled by a function P that satisfies the logistic differential equation

$$\frac{dP}{dt} = \frac{P}{5}\left(1 - \frac{P}{12}\right).$$

(a) If $P(0) = 3$, what is $\lim\limits_{t \to \infty} P(t)$?

If $P(0) = 20$, what is $\lim\limits_{t \to \infty} P(t)$?

(b) If $P(0) = 3$, for what value of P is the population growing the fastest?

(c) A different population is modeled by a function Y that satisfies the separable differential equation

$$\frac{dY}{dt} = \frac{Y}{5}\left(1 - \frac{t}{12}\right).$$

Find $Y(t)$ if $Y(0) = 3$.

(d) For the function Y found in part (c), what is $\lim\limits_{t \to \infty} Y(t)$?

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A certain rumor spreads through a community at the rate $\dfrac{dy}{dt} = 2y(1-y)$, where y is the proportion of the population that has heard the rumor at time t.

(a)   What proportion of the population has heard the rumor when it is spreading the fastest?

(b)   If at time $t = 0$ ten percent of the people have heard the rumor, find y as a function of t.

(c)   At what time t is the rumor spreading the fastest?

2006 AP® CALCULUS BC FREE-RESPONSE QUESTIONS (Form B)

5. Let f be a function with $f(4) = 1$ such that all points (x, y) on the graph of f satisfy the differential equation

$$\frac{dy}{dx} = 2y(3 - x).$$

Let g be a function with $g(4) = 1$ such that all points (x, y) on the graph of g satisfy the logistic differential equation

$$\frac{dy}{dx} = 2y(3 - y).$$

(a) Find $y = f(x)$.

(b) Given that $g(4) = 1$, find $\lim_{x \to \infty} g(x)$ and $\lim_{x \to \infty} g'(x)$. (It is not necessary to solve for $g(x)$ or to show how you arrived at your answers.)

(c) For what value of y does the graph of g have a point of inflection? Find the slope of the graph of g at the point of inflection. (It is not necessary to solve for $g(x)$.)

6. Consider the logistic differential equation $\frac{dy}{dt} = \frac{y}{8}(6 - y)$. Let $y = f(t)$ be the particular solution to the differential equation with $f(0) = 8$.

   (a) A slope field for this differential equation is given below. Sketch possible solution curves through the points $(3, 2)$ and $(0, 8)$.

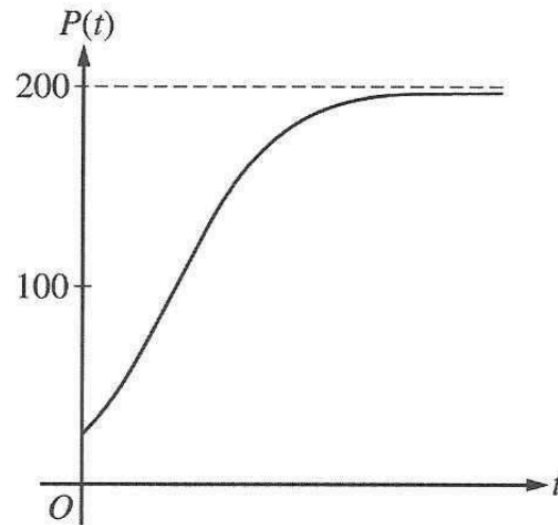   **(Note: Use the axes provided in the exam booklet.)**
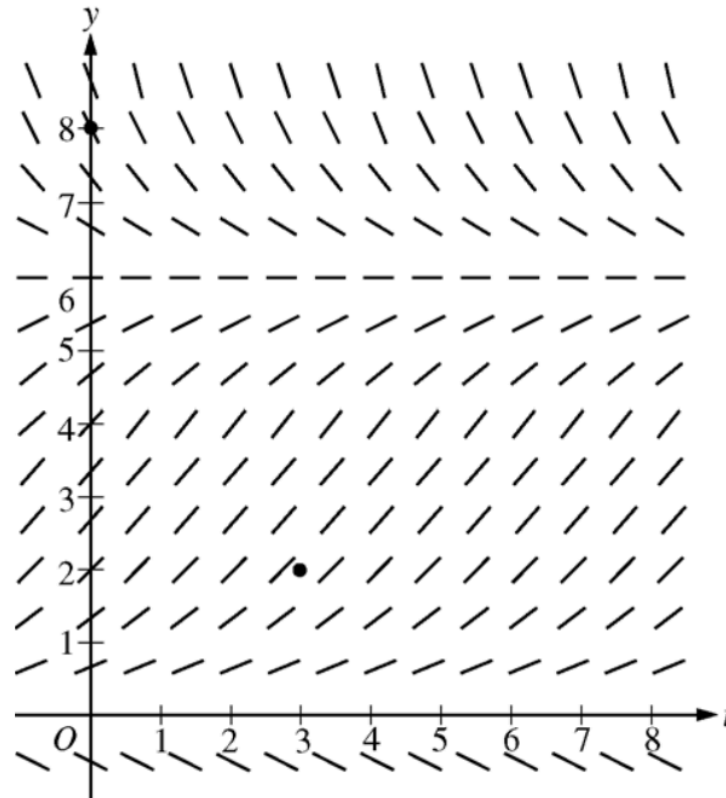
   (b) Use Euler's method, starting at $t = 0$ with two steps of equal size, to approximate $f(1)$.

   (c) Write the second-degree Taylor polynomial for f about $t = 0$, and use it to approximate $f(1)$.

   (d) What is the range of f for $t \geq 0$?

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24. Which of the following differential equations for a population P could model the logistic growth shown in the figure above?

(A) $\dfrac{dP}{dt} = 0.2P - 0.001P^2$

(B) $\dfrac{dP}{dt} = 0.1P - 0.001P^2$

(C) $\dfrac{dP}{dt} = 0.2P^2 - 0.001P$

(D) $\dfrac{dP}{dt} = 0.1P^2 - 0.001P$

(E) $\dfrac{dP}{dt} = 0.1P^2 + 0.001P$